定期試験問題　問題5

2)
$$I_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$I_x^2 = \frac{1}{2^2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{2^2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{2^2}E$$

$$I_x^3 = \frac{1}{2^3}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{2^3}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{2^3} I_x$$

$$\therefore\ I_x^{2n} = \frac{1}{2^{2n}}E, \quad I_x^{2n+1} = \frac{1}{2^{2n+1}} \times 2 \times I_x$$

$$e^{-i\alpha I_x} = \boxed{\sum_{n=0}^{\infty}(-1)^n\frac{1}{(2n)!}\left(\frac{\alpha}{2}\right)^{2n}}E - i\boxed{\sum_{n=0}^{\infty}(-1)^n\frac{1}{(2n+1)!}\left(\frac{\alpha}{2}\right)^{2n+1}} \times 2 \times I_x$$

$$= \cos\left(\frac{\alpha}{2}\right)E - i\sin\left(\frac{\alpha}{2}\right) \times 2 \times \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\left(\frac{\alpha}{2}\right) & 0 \\ 0 & \cos\left(\frac{\alpha}{2}\right) \end{pmatrix} - i\begin{pmatrix} 0 & \sin\left(\frac{\alpha}{2}\right) \\ \sin\left(\frac{\alpha}{2}\right) & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\left(\frac{\alpha}{2}\right) & -i\sin\left(\frac{\alpha}{2}\right) \\ -i\sin\left(\frac{\alpha}{2}\right) & \cos\left(\frac{\alpha}{2}\right) \end{pmatrix}$$

$$\therefore\ e^{i\alpha I_x} = \begin{pmatrix} \cos\left(\frac{\alpha}{2}\right) & i\sin\left(\frac{\alpha}{2}\right) \\ i\sin\left(\frac{\alpha}{2}\right) & \cos\left(\frac{\alpha}{2}\right) \end{pmatrix}$$

$$\therefore\ \rho_1 = \begin{pmatrix} \cos\left(\frac{\alpha}{2}\right) & -i\sin\left(\frac{\alpha}{2}\right) \\ -i\sin\left(\frac{\alpha}{2}\right) & \cos\left(\frac{\alpha}{2}\right) \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \cos\left(\frac{\alpha}{2}\right) & i\sin\left(\frac{\alpha}{2}\right) \\ i\sin\left(\frac{\alpha}{2}\right) & \cos\left(\frac{\alpha}{2}\right) \end{pmatrix}$$

$$= \begin{pmatrix} \cos\left(\frac{\alpha}{2}\right) & i\sin\left(\frac{\alpha}{2}\right) \\ -i\sin\left(\frac{\alpha}{2}\right) & -\cos\left(\frac{\alpha}{2}\right) \end{pmatrix}\begin{pmatrix} & \\ & \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\left(\frac{\alpha}{2}\right) - \sin^2\left(\frac{\alpha}{2}\right) & 2i\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right) \\ -2i\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right) & \sin^2\left(\frac{\alpha}{2}\right) - \cos^2\left(\frac{\alpha}{2}\right) \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} \cos\alpha & i\sin\alpha \\ -i\sin\alpha & -\cos\alpha \end{pmatrix}} \quad (2)$$

$\because\ \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$

$2\sin\alpha\cos\alpha = \sin 2\alpha$

$$=^{(3)} \boxed{\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}} \boxed{= -2I_y}$$

$$I_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$I_z^2 = \frac{1}{2^2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{2^2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{2^2} E$$

$$I_z^3 = \frac{1}{2^3}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{2^3}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{2^3} I_z$$

$$I_z^{2n} = \frac{1}{2^{2n}} E, \quad I_z^{2n+1} = \frac{1}{2^{2n+1}} \times 2 \times I_z$$

$$\rho_2 = e^{-i\omega I_z} \rho_1 e^{i\omega I_z}$$

$$e^{-i\omega I_z} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!}\left(\frac{\omega}{2}\right)^{2n} E - i\sum_{n=0}^{\infty}(-1)^n \frac{1}{(2n+1)!}\left(\frac{\omega}{2}\right)^{2n+1} \times 2 \times I_z$$

$$= \cos\left(\frac{\omega}{2}\right) E - i\sin\left(\frac{\omega}{2}\right) \times 2 \times I_z$$

$$= \begin{pmatrix} \cos\left(\frac{\omega}{2}\right) & 0 \\ 0 & \cos\left(\frac{\omega}{2}\right) \end{pmatrix} - i\sin\left(\frac{\omega}{2}\right)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\left(\frac{\omega}{2}\right) - i\sin\left(\frac{\omega}{2}\right) & 0 \\ 0 & \cos\left(\frac{\omega}{2}\right) + i\sin\left(\frac{\omega}{2}\right) \end{pmatrix}$$

又，$$e^{i\omega I_z} = \begin{pmatrix} \cos\left(\frac{\omega}{2}\right) + i\sin\left(\frac{\omega}{2}\right) & 0 \\ 0 & \cos\left(\frac{\omega}{2}\right) - i\sin\left(\frac{\omega}{2}\right) \end{pmatrix}$$

$$\rho_2 = \begin{pmatrix} \cos\left(\frac{\omega}{2}\right) - i\sin\left(\frac{\omega}{2}\right) & 0 \\ 0 & \cos\left(\frac{\omega}{2}\right) + i\sin\left(\frac{\omega}{2}\right) \end{pmatrix}\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}\begin{pmatrix} & \\ & \end{pmatrix}$$

$$= \begin{pmatrix} 0 & i\cos\left(\frac{\omega}{2}\right)+\sin\left(\frac{\omega}{2}\right) \\ -i\cos\left(\frac{\omega}{2}\right)+\sin\left(\frac{\omega}{2}\right) & \end{pmatrix} \begin{pmatrix} \cos\left(\frac{\omega}{2}\right)+i\sin\left(\frac{\omega}{2}\right) & 0 \\ 0 & \cos\left(\frac{\omega}{2}\right)-i\sin\left(\frac{\omega}{2}\right) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & i\cos^2\left(\frac{\omega}{2}\right)-i\sin^2\left(\frac{\omega}{2}\right)+2\sin\left(\frac{\omega}{2}\right)\cos\left(\frac{\omega}{2}\right) \\ -i\cos^2\left(\frac{\omega}{2}\right)+i\sin^2\left(\frac{\omega}{2}\right)+2\sin\left(\frac{\omega}{2}\right)\cos\left(\frac{\omega}{2}\right) & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & i\cos\omega+\sin\omega \\ -i\cos\omega+\sin\omega & 0 \end{pmatrix}$$

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$$= \boxed{\begin{pmatrix} 0 & \sin\omega+i\cos\omega \\ \sin\omega-i\cos\omega & 0 \end{pmatrix}} \quad (4)$$